

國立中央大學九十一年度轉學生入學試題卷

理、工、資電、地科學院 二年級 科目： 微積分 共 頁 第 頁

甲、填充題：每題 10 分。請將答案依題號寫在答案卷上，不必寫演算過程。

1. Find the volume _____ of the solid inside both

$$x^2 + y^2 + z^2 = a^2 \quad \text{and} \quad \left(x - \frac{a}{2}\right)^2 + y^2 = \left(\frac{a}{2}\right)^2.$$

2. If k is a positive integer, find the radius of convergence _____ of the series $\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn)!} x^n$.

3. Suppose g is the inverse function of a differentiable function f and let $G(x) = \frac{1}{g(x)}$. If $f(3) = 2$ and $f'(3) = \frac{1}{9}$. Find $G'(2) =$ _____.

4. Find the interval _____ on which the curve $y = \int_0^x \frac{dt}{1+t+t^2}$ is concave upward.

5. A conical paper cup 8 inches across the top and 6 inches deep in full of water. The cup springs a leak at the bottom and loses water at the rate of 2 cubic inches per minute. How fast is the water level dropping at the instant when the water is exactly 3 inches deep _____?

6. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$. Which of the functions are continuous at $(0, 0)$ _____?

(a) $f(x) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

(b) $f(x) = \begin{cases} \frac{\sin x - \sin y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

(c) $f(x) = \begin{cases} \frac{x^4y - y^5 + 4x^2y^3}{(x^2 + y^2)^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$

(d) $f(x) = (\cos^2(\pi\sqrt{x^2 + y^2}))^5$.

7. Find the area _____ of the region that consists of all points that lie within the circle $r = 2 \cos \theta$ but outside the circle $r = 1$.

乙、計算題：每題 15 分。須詳細寫出演算過程，否則不予計分。

1. Find the extreme value of

$$f(x, y) = x^2 + 2y^2 - 2x + 3$$

subject to the constraint $x^2 + y^2 \leq 10$.

2. Let $y = f(x)$ be a solution of the equation $y \sin x = x + \sin y$ for (x, y) near $(0, 0)$. Find the first three terms in the Taylor expansion of $f(x)$ about 0.

參考用